Adaptive evolution; concentrations in parabolic PDEs and constrained Hamilton-Jacobi equations

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Abstract

Living systems are subject to constant evolution through the two processes of mutations and selection, a principle discovered by Darwin. In a very simple, general and idealized description, their environment can be considered as a nutrient shared by all the population. This alllows certain individuals, characterized by a 'phenotypical trait', to expand faster because they are better adapted to the environment. This leads to select the 'best fited trait' in the population (singular point of the system). On the other hand, the new-born population undergoes small variance on the trait under the effect of genetic mutations. In these circumstances, is it possible to describe the dynamical evolution of the current trait?

We will give a mathematical model of such dynamics, based on parabolic equations, and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the 'best fitted trait' and eventually to compute various forms of branching points which represent the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the above mentioned parabolic equations one appropriately rescaled. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that desribe the population. We will show that a new type of Hamilton-Jacobi equation, with constraints, naturally describes this asymptotic. Some additional theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.

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