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“Small-scale dependent shock waves. Theory, approximation, and applications”

Abstract: In these lectures, I will investigate the existence and properties of small-scale dependent shock waves to nonlinear hyperbolic systems of partial differential equations, with an emphasis on

- (1) the theory of nonclassical entropy solutions involving undercompressive shock waves and
- (2) the theory of nonconservative hyperbolic systems that do not admit a divergence form.

Regularization-sensitive shock waves arise in continuum physics, especially in the dynamics of complex fluids and solid materials: multiphase (liquid-vapor) flows, turbulent flows, van der Waals phase transitions, austenite-martensite transformations.

To deal with undercompressive shock waves, the so-called kinetic relation (Truskinovsky, Abeyaratne, Knowles, LeFloch) was introduced as a mechanism to select the correct speed of the shock. On the other hand, the study of nonconservative hyperbolic systems is based on the DLM theory (Dal Maso, LeFloch, Murat) of nonconservative products in the setting of bounded variation functions. In the former, one prescribes a kinetic function while, in the latter, a family of Lipschitz continuous paths may be necessary. The role of a (single) entropy inequality is essential in these formulations and is tied to the regularization terms associated with a higher-order model, which takes into account additional physics and provides a description of small-scales.

In the last ten years, many analytical and numerical techniques were developed to study the effects of small scales in solutions to hyperbolic systems. I will begin the presentation with the Riemann problem

and the initial-value problem, using Glimm-type schemes and total variation functionals adapted to nonclassical entropy solutions. Then, the role of traveling wave solutions will be explained: in a given application, traveling waves (to the Navier-Stokes-Korteweg system, for instance) determine the relevant kinetic relation or the relevant family of paths. Furthermore, several applications will be discussed: generalized Camassa-Holm equations; model of thin liquid films; (hyperbolic-elliptic) equations of van der Waals fluids.

In short, these lectures will show that `several' shock wave theories are now available to encompass a variety of phenomena observed in complex fluid flows.